Mean value properties on Sierpinski type fractals

This is a joint work with Prof. Robert S. Strichartz.

For a domain Ω in Euclidean space, a function u is harmonic ($\Delta u = 0$) if and only if

$$\frac{1}{|B(x)|}\int_{B(x)} u(y)dy = u(x)$$

if $B_r(x) \subset \Omega$ where $B_r(x)$ is the ball of radius r about x. This is a mean value property of harmonic functions. There is a similar statement for mean values on spheres. More generally, if u is not assumed harmonic but Δu is a continuous function, then

$$\lim_{r\to 0}\frac{1}{r^2}(\frac{1}{|B_r(x)|}\int_{B_r(x)} u(y)dy - u(x)) = c_n \Delta u(x) \tag{0.1}$$

for the appropriate dimensional constant c_n.

What are the fractal analogs of these results? We do not want to specify in advance the nature of the sets on which we do the averaging. So if K is fractal and $x \in K$, we would like to know that there is a sequence of sets $B_k(x)$ containing x with $\cap_k B_k(x) = \{x\}$ such that

$$\frac{1}{\mu(B_k(x))}\int_{B_k(x)} u(y)dy = u(x)$$

for every harmonic function u. Moreover, for general u not assumed harmonic, we will show an analogous formula of (0.1).