Mean value properties on Sierpinski type fractals

This is a joint work with Prof. Robert S. Strichartz.

For a domain $\Omega$ in Euclidean space, a function $u$ is harmonic ($\Delta u = 0$) if and only if

$$\frac{1}{|B(x)|} \int_{B(x)} u(y) dy = u(x)$$

if $B_r(x) \subset \Omega$ where $B_r(x)$ is the ball of radius $r$ about $x$. This is a mean value property of harmonic functions. There is a similar statement for mean values on spheres. More generally, if $u$ is not assumed harmonic but $\Delta u$ is a continuous function, then

$$\lim_{r \to 0} \frac{1}{r^2} \left( \frac{1}{|B_r(x)|} \int_{B_r(x)} u(y) dy - u(x) \right) = c_n \Delta u(x)$$ (0.1)

for the appropriate dimensional constant $c_n$.

What are the fractal analogs of these results? We do not want to specify in advance the nature of the sets on which we do the averaging. So if $K$ is fractal and $x \in K$, we would like to know that there is a sequence of sets $B_k(x)$ containing $x$ with $\cap_k B_k(x) = \{x\}$ such that

$$\frac{1}{\mu(B_k(x))} \int_{B_k(x)} u(y) dy = u(x)$$

for every harmonic function $u$. Moreover, for general $u$ not assumed harmonic, we will show an analogous formula of (0.1).